



Sensitivity and elasticity in structured populations - Tutorial in spreadsheets

An important tool in the analysis of [matrix population models](#) is to understand how the transition and permanence probabilities of each class affect population growth. The amounts that express this are called sensitivity and elasticity. They are powerful tools both for the understanding of different life history strategies and for the management of threatened populations, or even for their sustainable use.

Sensitivity and elasticity measure the contribution of each element of the transition matrix to the composition of the growth rate (λ , or λ). Sensitivity measures the absolute contribution, while elasticity is a relative measure of that contribution.

In this exercise we will use a numerical method of perturbation of the transition matrix to calculate the contribution to the growth rate of each probability in the matrix. Basically, what we will do is vary slightly each of the values of the transition matrix at a time and see how the asymptotic growth rate (λ) changes. This method is called by some authors of “the easy brute force method”. There are more robust methods with more accurate answers, but the calculations are more complex and less intuitive (see section “[to learn more](#)”).

Let's apply perturbations to the same matrices used in the script of [introduction to transition matrices](#). They are real data of cactus populations *Escobaria robbinsorum* and of the palm heart *Euterpe edulis*.

Elasticity and Sensibility of a *Escobaria robbinsorum* population

- **1.** Download and open the spreadsheet for the [elasticity analysis](#) of *Escobaria robbinsorum*. In this worksheet, we have at the beginning of the original transition matrix. To make sure you are viewing this matrix, scroll to the A1 cell of the worksheet. Then we have the “matrix for population projection and λ calculation”, in which [the procedure used to calculate the growth rate](#) (λ) from of population projection is already implemented. For your convenience, note, beside the original transition matrix, the λ value obtained with the original values.
- **2.** In the “Matrix for population projection and λ calculation”, it produces a small disturbance in the probability of permanence in class 1 (seedling → seedling) of 0.001. That is, modify the cell value by adding 0.001. Note that by modifying the value of this cell, the projection values will change. Check to see if there is a stabilization of the λ value. The stabilized value is the new growth rate (λ) of the population from the disturbance produced.
- **3.** Sensitivity Calculation: To the right of the “Sensitivity Matrix” are the information you need

to calculate the sensitivity and at the end a cell with the formula already prepared, based on the following equation ¹⁾:

$$S_{1,1} = \frac{\lambda_{\text{pert}} - \lambda_{\text{orig}}}{a_{\text{pert}(1,1)} - a_{\text{orig}(1,1)}}$$

- Before you begin calculating, look at the formulas in the worksheet and make sure you are understanding what is being calculated. The value calculated for sensitivity when modifying class 1 (seedling→ seedling) should be copied to cell 1 of the “Sensitivity Matrix” using the **EDIT/PASTE SPECIAL/VALUES**.
- **4.** Repeat this procedure for all transition probabilities (ie, seedling→ juvenile, seedling→ adult, etc.) and complete your Sensitivity Matrix.
- **TIP: Remember that you must disturb one element of the array at a time. The others are left with their original value. Then, when repeating the procedure for the next transition probability, it is important to return the value of the cell being analyzed before the original value. To make sure that you are only modifying the value that matters at the moment, use the original matrix at the top of the worksheet to see if the other values are the same.**
- Check which of the transition probabilities produces the greatest effect on the population growth rate (λ).
- **5.** For the calculation of the elasticity it is only to divide each difference of the formula above by the original value so that the differences are expressed in proportion. An alternative is to multiply the Sensitivity value by the ratio between the original rate and the original λ :

$$E_{ij} = \frac{a_{\text{orig}(1,1)}}{\lambda_{\text{orig}}} S_{ij}$$

Portanto, a elasticidade é:

$$E_{ij} = \frac{a_{\text{orig}(ij)}}{\lambda_{\text{orig}}} S_{ij}$$

- **6.** Then, finally, in the “Matrix of Elasticity”, mount, for each cell, a formula to calculate the Elasticity, according to the equation indicated above. Now check which cell produces the greatest effect on population growth rate (λ).

Elasticity and Sensibility of the palm heart population

- **1.** Download the spreadsheet for [elasticity analysis of the palm heart](#) (data from Silva Matos et al. 1999).
- **2.** Find out a vulnerable spot in this palm heart population. Choose **two** elements of the palm transition matrix that you consider the most important and the least important for the viability of this population. Calculate the sensitivity and elasticity **of these two elements** and verify if the values confirm your suspicion.

To learn more

Basic bibliography

Caswell, H. 2001. Matrix Population Models (Second edition), Sinauer Associates, Sunderland.

Freckleton, R.P., Silva Matos, D.M., Bovi, M.L.A & Watkinson, A.R. 2003. Predicting the impacts of harvesting using structured population models: the importance of density-dependence and timing of harvest for a tropical palm tree. *Journal of Applied Ecology*, 40: 846-858.

Gotelli, N. J. 2007. *Ecologia*. Cap.3- Crescimento Populacional Estruturado. Pp. 49-82. Ed. Planta.

Gurevitch, J, Scheiner, S.M, Fox, G.A. 2009. *Ecologia Vegetal*. Cap. 5 - Ed. Artmed, São Paulo.

Silva Matos, D.M., Freckleton, R.P. & Watkinson, A.R. 1999. The role of density dependence in the population dynamics of a tropical palm. *Ecology*, 80: 2635-2650.

Application examples

[Hal Caswell](#) is the main researcher in the area of matrix models in Ecology. His [book](#) is the basic reference about it. Another good example of applications you can find [here](#).

Stubben, C., & Milligan, B. (2007). Estimating and analyzing demographic models using the popbio package in R. [Journal of Statistical Software](#), 22(11), 1-23.

[planilha](#), [uma população](#), [população estruturada](#)

1)

the mathematical definition of λ is the derivative of λ with respect to a_{ij} . The following equation is a numerical approximation that works with small perturbations. To learn more, see the section [To learn more!](#) 😊

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