-
- Exercicios

Integration

Exercise 1

Figure out the result of the following integrals:

a) $ \int \sin(x) dx$

b) $ \int x^2 +1 dx$

c) $ \int _0^1 \cos (x) dx$

d) $ \int _{-1}^5 x^3 +2x dx$

e) $ \int _1 ^ \infty \frac{1}{x^2}dx$

f) $ \int _0 ^ 1 \sin (x^{27}) dy $ (beware, this is a trick question!)

Which of the above are definite integrals, and which are indefinite?

Exercise 2

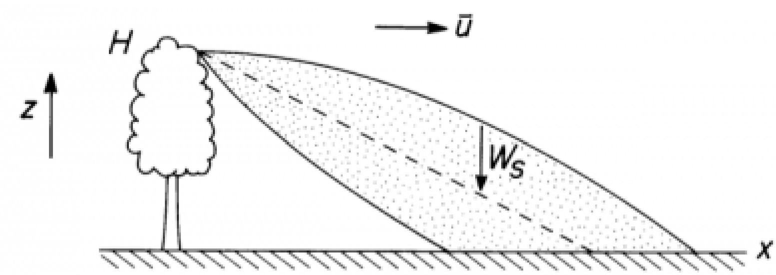
(Use Maxima)

In some species, the main dispersion agent is the wind. It is possible to model the distribution of seeds over distance mechanistically. One expression, that can be used for winds in one direction is:

- $ Q(x) = \frac{NW_s}{\sqrt{2\pi}\bar u \sigma_z} \exp \left[- \frac{ (H-W_sx/\bar u)^2 } {2 \sigma_z^2} \right] $

The parameter of this equation are:

- N: rate of seed generation in the source
- σ_z: vertical variance in the seed random walk
- W_s: fixation speed for the seeds
- $\bar u$: average speed for the wind
- H: source height



(Okubo & Levin, 1989)

1) What is the total of seeds that one tree disperses over a 1m radius? To answer this, integrate the function $Q(x)$ from -1 to 1. Use $N = 100$, $\sigma_z = W_s = \bar u = H = 1$.

2) What is the total of seeds dispersed over the entire x axis? Does this result surprise you?

3) What is the expression that, for a given distance d, gives the number of seeds dispersed from 0 to d?

- (Note: ERF? Read about this weird function here)

Exercise 3

In the exercise 2, we have considered the dispersal of seeds over the space in a fixed time (like a photo). Let's observe the production of seeds over a whole year, and N will be a periodical function over time to represent the different seasons:

* $ N(t) = N_0(\sin(t) + 1) $

Let's use $N_0 = 100$, so our previous exercise corresponds to the case$\sin(t)=0$ (for example, $t=0$).

Now, our function $Q(x, t)$ depends not only on x, but also on t:

$ Q(x, t) = \frac{N(t)W_s}{\sqrt{2\pi}\bar u \sigma_z} \exp \left[- \frac{ (H-W_sx/\bar u)^2 } {2 \sigma_z^2} \right] $

1) Find out what is the density of seeds dispersed between -1 and 1 at time $t=0$, to make sure that you find the same result as before: $\int _{-1}^1 Q(x, 0) dx$.

2) Find out the density of seeds that fall over the point $x=1$ over a full year, that is, with t varying from 0 to 2π. To do so, solve the integral *over time*: $ \int _0 ^ {2 \pi} Q(1, t) dt$.

3) Figure out one expression for the total density of seeds in a moment t, on the radius from -1 to 1. Note that this answer will be a function of t, so we will call it $h(t) = \int _{-1}^1 Q(x,t) dx$.

4) Use the function you wrote down on question 3 to find the total number of seeds dispersed between -1 and 1 during one year.

In this last exercise, you have calculated the integral $ \int _0 ^ {2 \pi} h(t) dt$. If you write in full the definition of $h(t)$, we will have:

$\int _0 ^ {2 \pi} \int _{-1} ^ 1 Q(x, t), dx dt$

Congratulations! You have just completed a multiple integration!

Challenge

(You don't need to turn over this section, but read it with care ;))

1) In the exercise 2, the source of the seeds is fixed on the origin (that is, position 0). If the tree is at a generic position x, what is the expression that gives the seed falling rate in a point y?

2) Let's change our point of view. In an expedition across the x axis, the intrepid explorers have found a dense forest, extending from point A to point B. Suppose that the forest is composed of N seed sources distributed homogeneously. Can you write a function that gives the seed falling rate for a given point in this axis? (suppose that the wind direction is from A to B)

3) Is it possible to measure the emigration of this forest? (the rate of seeds that fall after point B)

4) What would change in the expression you wrote for 2 if the forest had the seeds distributed according to a normal distribution?

5) And what if the wind came from both sides with equal probability?

6) What is the point in which the largest number of the seeds will fall?

Answer

See the exercise solutions: solexintegral.wxm